

③ a)

F1	F2	Color	Result
L	H	R	good
L	L	B	bad
H	H	B	good
L	H	B	bad
H	L	R	good

b)  $S = [3+, 2-] ; p^+ = .6, p^- = .4$

$$E(S) = -.6 \log_2 .6 - .4 \log_2 .4$$

$$\approx 0.971$$

F1

$$S_{F1=H} = [2+, 0-]$$

$$S_{F1=L} = [1+, 2-]$$

$$E(S_{F1=H}) = -\log_2 1 = 0$$

$$E(S_{F1=L}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = .719$$

$$IG(S, F1) = E(S) -$$

$$\left( \frac{S_{F1=H}}{S} \right) E(S_{F1=H})$$

$$- \left( \frac{S_{F1=L}}{S} \right) E(S_{F1=L})$$

$$= .971 - 0 - \frac{2}{3} (.719)$$

$$= .540$$

$S_{Fa}$

$$S_{Fa=H} = [2^+, 1^-]$$

$$S_{Fa=L} = [1^+, 1^-]$$

$$E(S_{Fa=H}) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = .719$$

$$E(S_{Fa=L}) = 2 \left( -\frac{1}{2} \log_2 \frac{1}{2} \right) = .890$$

$$\begin{aligned} IG(S, Fa) &= .971 - \left( \frac{3}{5} \right) .719 - \left( \frac{2}{5} \right) .890 \\ &= .184 \end{aligned}$$

$S_{Color}$

$$S_{Color=R} = [2^+, 0^-]$$

$$S_{Color=B} = [1^+, 2^-]$$

$$S_{Color=Y} = [0^+, 0^-]$$

$$E(S_{Color=R}) = -1 \log_2 1 = 0$$

$$E(S_{Color=B}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = .719$$

$$E(S_{Color=Y}) = 0$$

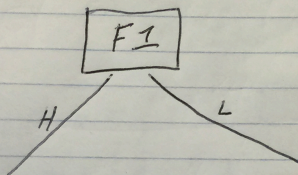
$$\begin{aligned} IG(S, Color) &= .971 - \left( \frac{2}{5} \right) 0 - \left( \frac{3}{5} \right) .719 - \left( \frac{0}{5} \right) 0 \\ &= .540 \end{aligned}$$



Feature	IG
F1	.544
F2	.184
Color	.544

Root node is F1:

- Highest IG Value
- Tie broken over Color



F2	Color	Result
H	B	good
L	R	good

F2	Color	Result
H	R	good
L	B	bad
H	B	bad

~~F2=H~~

~~$$S(F1=H, F2=H) = S(F1=H, F2=L) = [1^+, 0^-]$$~~

~~$$S(F1=H, F2=H) = E(S(F1=H, F2=L)) = -1 \log_2 1 = 0$$~~

~~$$S(S, [F1=H, F2]) = .971 - 0 - 0 = .971$$~~

Yes

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$$E(S_{F1=L}) = \frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = .918$$

$$S_{F1=L, F2=H} = [1^+, 1^-]$$

$$S_{F1=L, F2=L} = [\emptyset^+, 1^-]$$

$$E(S_{F1=L, F2=H}) = 2 \left( -\frac{1}{2} \log_2 \frac{1}{2} \right) = .890$$

$$E(S_{F1=L, F2=L}) = -1 \log 1 = 0$$

$$IG(S_{F1=L, F2}) = E(S_{F1=L}) - \frac{E(S_{F1=L, F2=H}) \cdot P(F2=H)}{3} - 0$$

$$= .918 - \frac{.890 \cdot \frac{2}{3}}{3} = .826$$

$$S_{F1=L, Color=R} = [1^+, \emptyset^-]$$

$$S_{F1=L, Color=B} = [\emptyset^+, \emptyset^-]$$

$$S_{F1=L, Color=Y} = [\emptyset^+, \emptyset^-]$$

$$E(S_{F1=L, Color=R}) = 0 \quad (= E(S_{F1=L, F2=L}))$$

$$E(S_{F1=L, Color=B}) = 0 \quad (= E(S_{F1=H}))$$

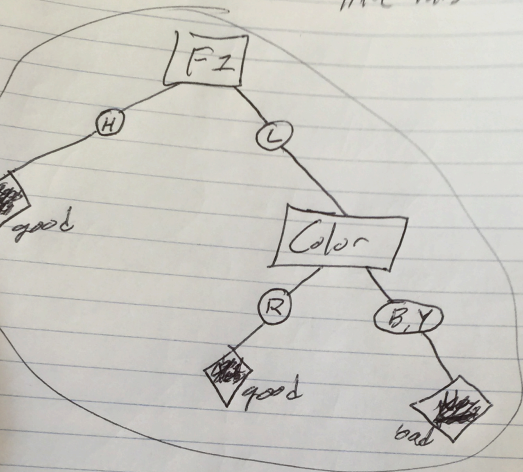
$$E(S_{F1=L, Color=Y}) = 0 \quad (\text{nothing to choose from})$$

$$IG(S_{F1=L, Color}) = .918 - 0 - 0 - 0$$

$$= .918$$

highest is Color,  
use that as next node

o, final tree looks like this:



The prediction accuracy can be determined by running it through the tree and calculating the number of correct results:

F1(H/L)	F2	F2(H/L)	Color	Result	Tree Result
L	4	H	R	good	good
L	0	L	B	bad	bad
H	7	H	Y	good	good

because all results match the results from the tree, the prediction accuracy can be determined to be 100%.